Mathematica Calculations

Constants:

In[1]:=
```
me := 9.10938356*^-31;      (* Mass of Electron kg *)
mD := 3.343583719*^-27;     (* Mass of Deuterium kg *)
mT := 5.00736*^-27;         (* Mass of Tritium kg *)
```
$\mu := \frac{mD * mT}{mD + mT};$ (* Reduced Mass kg *)
```
e := 1.60217662*^-19;       (* Charge of Proton C (i.e. elementary charge) *)
```
$\epsilon_0 := 8.854187817*\wedge\text{-}12;$ (* Permittivity of Free Space $\frac{C^2}{J\ m}$ *)

$\hbar := 1.054571800*\wedge\text{-}34;$ (* Planck's Constant J s *)

k := 1.3807*^-23; (* Boltzmann Constant $\frac{J}{K}$ *)
```
T := 3*^8;                  (* Plasma Temperature K (From Lawson Criteria) *)
```
c := 2.9979*^8; (* Speed of Light $\frac{m}{s}$ *)
```
(* These are constants defined in and
 calculated from simulation density and temperature *)
```
$\lambda_D := 5.350161*\wedge\text{-}5;$ (* Debye Length of Simulation in m *)
```
PP := 7.936442*^-13;        (* Plasma Period in s *)
```
$\text{SimLength} := 10 * \lambda_D;$ (* Entire Length of simulation in m *)
```
Num := 5*^5;                (* Number of particles per species *)
```
$V_{Drms} := 2.647617*\wedge\text{-}2$ (* Deuterium rms Velocity in $\frac{\lambda_D}{PP}$ *)

$V_{Trms} := 2.171260*\wedge\text{-}2$ (* Tritium rms Velocity in $\frac{\lambda_D}{PP}$ *)

Tunneling Constants, E_0 and R_0:

In[17]:=
$$E0[m\_] := \left(\left(\frac{e^2}{4\,\pi\,\epsilon_0}\right)\frac{\pi}{2}\,\frac{\sqrt{2\,m}}{\hbar}\right)^2;$$

$$R0[m\_] := \frac{1}{\left(\left(\frac{e^2}{4\,\pi\,\epsilon_0}\right)^{1/2}\frac{2}{\hbar}\,\sqrt{2\,m}\right)^2};$$

Finding the tunneling constants using the Deuterium - Tritium reduced mass:

In[19]:=
```
E0[μ] (* Result in Joules *)
R0[μ] (* Result in meters *)
```

Out[19]= 4.73508×10^{-14}

Out[20]= 3.00549×10^{-15}

Finding Nuclear Radius, r_0:

In[21]:=
```
r0[A_] := 1.25 A^(1/3)          (* where A is the number of neutrons plus protons *)
```

The result of a Deuterium-Tritium fusion will have 2 protons and 3 neutrons, so A=5

In[22]:=
```
r0[5] (* Result in femtometers *)
```

Out[22]= 2.13747

Lorentz Factor

In[23]:= $\gamma[v\_] := \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

The fastest Deuterium-Tritium relative velocity recorded corresponds to a simulation running with an energy of 1000 keV: $0.2800237 \, \frac{\text{debye lengths}}{\text{plasma period}}$

In[24]:= Velocity := $0.2800237 * \frac{\lambda_D}{PP}$

Velocity (* Result in $\frac{m}{s}$ *)

Out[25]= 1.88771×10^{7}

In[26]:=
```
γ[Velocity] (* Result is unitless *)
```

Out[26]= 1.00199

Maxwell-Boltzman Distribution

The Thermal deBroglie wavelength for various particles and the inverse one-dimensional density η_{1D}^{-1}:

In[27]:= $\lambda_{th}[m\_] := \frac{2\pi\hbar}{\sqrt{2\pi m k T}}$; (* in meters *)

$\lambda_{th}[me]$

$\lambda_{th}[mD]$

$\lambda_{th}[mT]$

$\lambda_{th}[\mu]$

Out[28]= 4.3034×10^{-12}

Out[29]= 7.10313×10^{-14}

Out[30]= 5.80432×10^{-14}

Out[31]= 9.17304×10^{-14}

In[32]:= $\eta_{1D} = \frac{4 * Num}{SimLength}$; (* in meters^{-1} *)

η_{1D}^{-1}

Out[33]= 2.67508×10^{-10}

In[34]:= Cos[90]

Out[34]= Cos[90]

An Appropriate Timestep

Example: Δt 1D vs 3D

For a total of 1000 Deuterium and Tritium ions and an average rms velocity of

In[35]:= $v_{rms} := \frac{V_{Drms} + V_{Trms}}{2}$; (* Debyle length per plasma period *)

One-Dimensional (1 Debye length)

In[36]:= $\eta_{1D} := \frac{1000}{1}$ (* Particles per Debye length *)

$\Delta t_{1D} = \frac{1}{v_{rms}\,\eta_{1D}}$ PP (* units scaled to Seconds *)

Out[37]= 3.2939×10^{-14}

Three-Dimensional (1 Debye length)^3

In[38]:= $\eta_{3\,D} := \eta_{1\,D}{}^3$ (* Particles per Debye length *)

$\Delta t_{3\,D} = \frac{1}{v_{rms}\,\eta_{3\,D}}$ PP (* units scaled to Seconds *)

Out[39]= 3.2939×10^{-20}